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Flight Control Design Based on Nonlinear Model with Uncertain Parameters

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A simplified but significantly nonlinear model is used for the short-period longitudinal flight control problem, with c^* taken as the output. Uncertainty is included in the model by allowing for a large range in velocity and air density, without any provision for their measurement. The output $c^*(t)$ is to lie within specified bounds in response to a range of step commands. A recent exact design technique for uncertain nonlinear systems is used. In this technique, the nonlinear plant set is replaced by a linear time-invariant plant set, which is precisely equivalent to the nonlinear set with respect to the problem tolerances. The design execution then involves frequency response concepts and techniques. For a large problem class, the resulting design is guaranteed to solve the problem for the specific class of inputs posed. The design was simulated with excellent results.

Nomenclature

a	= parameter in model of $T(s)$
$B(\omega)$	= bounds on $G(j\omega)$
C	= mean aerodynamic chord, m
$C_{ij}(\alpha)$	= various aerodynamic coefficients
c^*	= output variable, a handling qualities criterion
$C^*(s)$	$= \mathcal{L}c^*(t)$
e_G	= excess of poles over zeros of $G(s)$
$F(s)$	= prefilter transfer function
$G(s)$	= loop compensation function
I_y	= moment of interia in pitch, kg-m^2
j	$= \sqrt{-1}$
k	= see R
lti	= linear time invariant
$L(s)$	$= G(s)P(s)$ = loop transfer function
$\mathcal{L}$	= Laplace transform
m	= mass of vehicle, kg
mp	= minimum-phase
nmp	= non-minimum-phase
$p(t), p_i$	= equivalent lti plant function
$P(s)$	$= \mathcal{L}p(t)$
$\mathcal{P}$	= set $\{p\}$ or $\{P(s)\}$
$q=\dot{\theta}$	= pitch angular velocity
$r(t)$	= system command input
$R(s)$	$= \mathcal{L}r(t) = k/s$
$\mathcal{R}$	= set $\{r(t)\}$ or $\{R(s)\}$
s	= complex variable
S	= wing surface area, m^2
$T(s)$	= closed loop transfer function
$\mathfrak{T}$	= set $\{T(s)\}$
$\mathfrak{T}_p$	= template of $P = P(j\omega)$
u	= vertical velocity, m/s
v	= horizontal velocity, V_0 constant
w	= nonlinear plant function, $y = w(x)$
w^{-1}	= inverse of w, $x = w^{-1}(y)$
$\mathcal{W}$	= set $\{w\}$
x, x_i	= input to nonlinear plant
$X(s)$	$= \mathcal{L}x(t)$
y, y_i	= acceptable output of plant
$Y(s)$	$= \mathcal{L}y(t)$
$\mathcal{Y}$	= set $\{y\}$
α	= angle of attack
δ	= elevator deflection
$\Delta(s)$	$= \mathcal{L}\delta(t)$
ζ	= parameter in model of $T(s)$
ρ	= air density, kg/m^3
ω	= frequency, rps

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Introduction

IN most flight control design techniques, the nonlinear differential equations are linearized about a trim condition. The resulting incremental linear time invariant (lti) model, with its fixed aerodynamic coefficients, is reasonably valid for small excursions from trim. By using different trim points, there are obtained different sets of coefficient values. The model is then taken as lti, in which these sets are treated either 1) as uncertain parameters in an lti design technique for securing specified performance over the uncertain parameter set, or 2) as known functions of Mach and dynamic pressure (the latter of which are monitored, leading to a "scheduling" design), or 3) as uncertain parameters to be identified and compensated for in a so-called "adaptive" design. A combination of all three may also be used. However, in all of these approaches the model used is lti.

There have recently been attempts at incorporating the nonlinearities to some extent into the system model, the vehicle for doing so being optimal control theory. However, the calculations have been very laborious, requiring considerable approximations (see Ref. 1 and its references for discussion). This paper takes a far different approach and is based on a recent[2] synthesis technique for feedback around a nonlinear uncertain plant, the latter denoting the constrained part of the system. This synthesis technique has the following important properties:

1) For a large class of practical nonlinear plants, the design is precise and direct with no approximations even for highly nonlinear (even nonlinear time-varying) models with large uncertainties in the plant parameters.

2) Design execution is in the frequency domain.

Review of Nonlinear Design Technique

The key ingredient is the extension to sets of an idea often used by engineers: Given a specific nonlinear element w, it is usually possible to find an lti element p which is equivalent, in a certain sense, to w for a single specific output $y_1(t)$ (or input $x_1(t)$). Let $y_1(t) = w[x_1(t)]$ be the (assumed) *unique* output of w due to input $x_I(t)$. Choose p so that its output is also y_I when its input is $x_I(t)$. A simple way to do this is to find (Laplace transform) $\mathcal{L}y_I(t) \triangleq Y_I(s)$ and $\mathcal{L}x_I(t) \triangleq X_I(s)$. Then let $P(s) \triangleq Y_I(s)/X_I(s)$, giving $p(t) = \mathcal{L}^{-1}P(s)$ the lti equivalent of w, but only for the special case when the input is x_I. Hence, the two essential conditions are: 1) w has a unique inverse, thus excluding hard saturation etc. which, however, could be replaced by very low gain over the appropriate interval; 2) the desired plant output $y(t)$ and the resulting $x = w^{-1}(y)$ are Laplace transformable, a condition difficult to violate.

This equivalence idea can be extended to a set of nonlinear plants $\mathcal{W} = \{w\}$ for which an lti set denoted by $\mathcal{P}_I$ can be found which is equivalent to $\mathcal{W}$, with respect to $y_I(t)$. Simply find p_i, the y_I-equivalent of w_i, for each $w_i \in \mathcal{W}$, and let $\mathcal{P}_I = \{p_i\}$. A further extension is to find an lti set $\mathcal{P}$ which is equivalent to $\mathcal{W}$ with respect to a *set* of outputs $\mathcal{Y} = \{y_j\}$. To find $\mathcal{P}$, repeat the previous for each $y_j \in Y$ giving $\mathcal{P}_j$ and then $\mathcal{P} = \{\mathcal{P}_j\}$. If $\mathcal{W}$ has 10 elements and $\mathcal{Y}$ has 20, $\mathcal{P}$ has 200 members. But in general, both $\mathcal{W}$ and $\mathcal{Y}$ are uncountable and so is $\mathcal{P}$.

In Fig. 1, the closed-loop system is to behave like an lti one, with transfer function $T(s)$, in response to any command input in a set $\mathcal{R} = \{r\}$ and for any $w = \mathcal{W}$, i.e., $\mathcal{L}y(t) \triangleq Y(s) = T(s)R(s)$, where $R(s) = \mathcal{L}r(t)$. However, due to the uncertainty, a set of acceptable $\{T(s)\} = \mathfrak{I}$ must be specified, insomuch as dynamic invariant response for all w in $\mathcal{W}$ is impossible. The sets $\mathcal{R}$, $\mathfrak{I}$ determine the set of desired acceptable system outputs $\mathcal{Y} = \{y(t)\}$ via $\mathcal{L}y(t) = Y(s) = T(s)R(s)$, or vice versa. The nonlinear set $\mathcal{W}$ is then replaced by the lti set $\mathcal{P}$ which is equivalent to $\mathcal{W}$ with respect to $\mathcal{Y}$. We say that $\mathcal{P}$ is mp (minimum-phase) with respect to $\mathcal{Y}$, it all $P(s) \in \mathcal{P}$ have no zeros in the right half-plane, and stable if they all have no such poles. In our specific problem $\mathcal{P}$ is mp, but not stable.

We now have a pure lti feedback problem: Find the lti compensation $F(s)$, $G(s)$ needed in Fig. 1, such that the system transfer function $T = FL/(1+L)$, $L = GP$ is a member of the set $\mathfrak{I}$, no matter which $P \in \mathcal{P}$ is used. A frequency-response design technique for a very general problem class,[3] is available for minimum-phase $\mathcal{P}$. $\mathcal{P}$ may have unstable elements. If this lti problem is solvable, then under quite general conditions the solution (i.e., the $F(s)$, $G(s)$ compensation pair used) is also precisely valid for the original nonlinear problem. The details and proof[2] involve functional analysis techniques. However, design execution involves frequency-response concepts, as next seen.

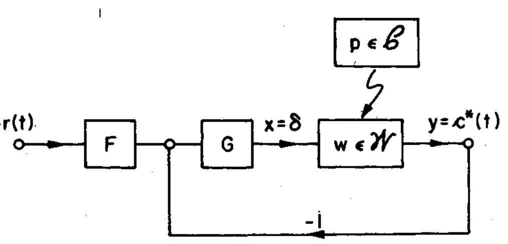


Fig. 1 System structure—$\mathcal{W} = \{w\}$ is replaced by $\mathcal{P} = \{p\}$.

Problem Statement and Execution

Nonlinear Plant

The nonlinear plant is described by Eqs. (1-3):

$$\dot{u} = qV_0 + g\cos\theta - (\rho V_0 S/2m)\left[C_{N\alpha}(\alpha) + C_{N\delta}(\alpha)\delta\right] \quad (1)$$

$$\ddot{\theta} = \dot{q} = \frac{\rho V_0 SC}{2I_y}\left[C_{m\delta}(\alpha)\delta + C_{M\alpha}(\alpha) + \frac{C}{2V_0}C_{mq}(\alpha)q\right] \quad (2)$$

$$\alpha = \tan^{-1}(u/V_0) \quad (3)$$

$$c^* = 12.4\dot{\theta} + (\dot{u} - V_0\dot{\theta} + 6\ddot{\theta})/9.8 \quad (4)$$

Eq. (4) is the output[4] to be controlled, i.e. to be in $\mathcal{Y}$. The numbers used are[5,6] $I_y = 207{,}000$ kg-m^2, $m = 17{,}600$ kg, $C = 4.89$ m, $S = 49.2$ m^2. The $C_{ij}(\alpha)$ are nonlinear functions of α (see Fig. 2). Since simulated α ranges in [0,35 deg], there is strong nonlinear operation. The horizontal velocity v was taken as V_0 fixed, which is incorrect for some low-velocity cases, but the objective here is to demonstrate the validity of the design technique in a strongly nonlinear situation, which is achieved sufficiently by means of the nonlinear $C_{ij}(\alpha)$.

The bounds[4] on the acceptable $c^*(t)$ in response to a unit step command are included in Fig. 3, which also includes design simulation results. The set of command inputs $\mathcal{R}$ consists of steps 1 to 5 in magnitude. Parameter uncertainty is due to ρ ranging in [0.3,1.22] and V_0 in [75,206]. Initial conditions are $\dot{w}(0) = \dot{q}(0) = q(0) = 0$, $\alpha(0) = \theta(0)$, giving initial values for δ (as well as u, θ, α) which is subtracted out so that the change in δ is used to find the lti equivalent $\mathcal{P}$ set. The detailed steps in the design implementation are presented next, with comments postponed to the end.

Step 1: The lti Set $\mathcal{P}$

Let $\mathcal{L}c^*(t) \triangleq C^*(s) = T(s)R(s)$, $R(s) = k/s$ with $k \in [1,5]$ and $T(s) \in \mathfrak{I}$, derived from the bounds in Fig. 3. A simple means for generating $\mathfrak{I}$, taken from Ref. 5, is to let

$$T(s) = a^2(s+2.9)/2.9(s^2 + 2\zeta as + a^2)$$

ζ ranging in [.7, 1.5], a in [3.14, 7.6], giving the bounds on $|T(j\omega)|$ in Fig. 4. Such bounds suffice[3] for mp (and obviously stable) $T(s)$. Any $T(s)$, $R(s)$ pair thus generates an acceptable $c^*(t)$. A computer program solved Eqs. (1-4) backwards for $\delta(t)$ and then checked the result by solving Eqs. (1-4) forwards for $c^*(t)$ from $\delta(t)$. The program was considered adequate only when excellent agreement was obtained over the entire range considered. $C^*(s)$ was a priori

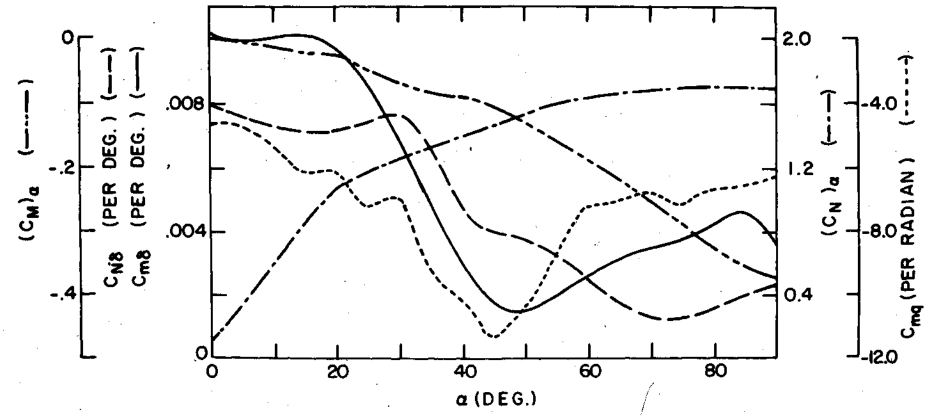


Fig. 2 Aerodynamic coefficients $C_{ij}(\alpha)$.

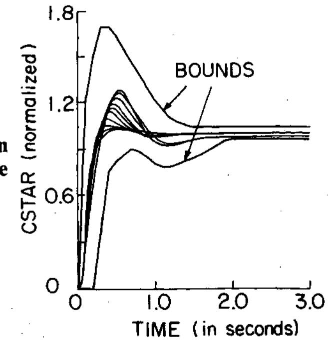


Fig. 3 Normalized bounds on acceptable $c^*(t)$—representative simulation results.

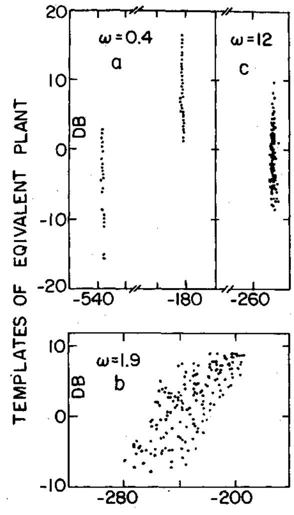


Fig. 6 Templates $\mathfrak{J}p(\omega)$ of $\{P(j\omega)\}$ at $\omega=0.04, 1.9, 12$.

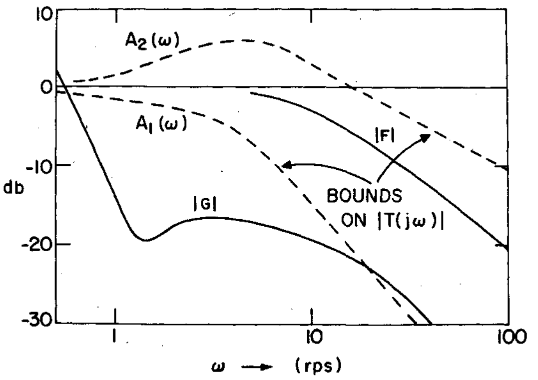


Fig. 4 Bounds on $|T(j\omega)|$—Bode plots of designed $|F(j\omega)|$, $|G(j\omega)|$.

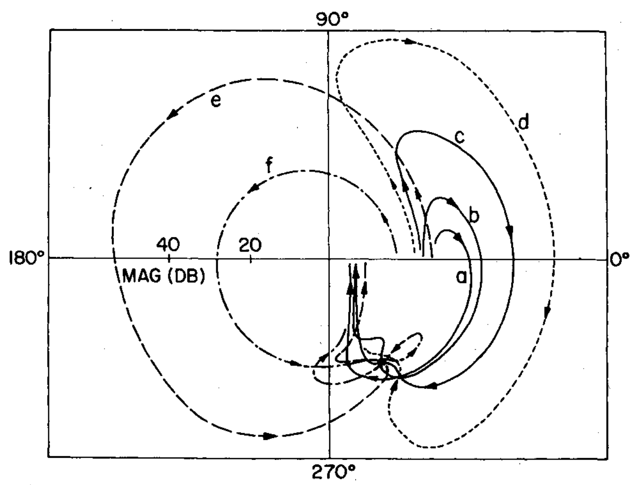


Fig. 5 Loci of typical lti $P(j\omega)$—cases e and f are open-loop unstable. See Table 1.

available and $\Delta(j\omega) \triangleq \mathfrak{L}\delta(t)]_{j\omega}$ was obtained by numerical integration. As c^* is a short-period criterion and for all the acceptable cases has definitely reached steady-state in 4 s (see Fig. 3), Eqs. (1-4) were solved only for $t\in[0,4\text{ s}]$, and the constant $\delta(4)$ was used for $t>4$.

Loci of six $P(j\omega)$ described in Table 1 are shown in Fig. 5. Two of them (e,f) are unstable with a pair of right half-plane poles, which are zeros of $\Delta(s)$. The set includes a large number of such unstable lti $P(s)$, which the lti design technique[3] can easily handle.

Plant Templates

At any ω say $\omega=\omega_l$, the set $\{P(j\omega)\}$, $P\in\mathcal{P}$ consists of a region in the logarithmic complex plant (Nichols chart) denoted as the ω_l-plant template $\mathfrak{J}p(\omega_l)$. A number of $\mathfrak{J}p(\omega)$ are shown in Fig. 6. At very small ω there are two almost constant angle subtemplates 360 deg apart. This is because the presence of both stable and unstable $P\in\mathcal{P}$, and the fact that Arg P near $\omega=0$ is either ± 0 or $n\pi/2$ for some integer n. As ω increases, the two groups merge together and approach a single vertical line at large ω well beyond the plant "dynamics." (In this case, $\omega=12$ is large enough, see Fig. 6.) Note how this frequency response approach is indifferent to system order.

Step 2: Bounds $B(\omega)$ on $G(j\omega)$

Given the set $\mathcal{P}=\{P\}$, the problem is to find $F(s)$, $G(s)$ in Fig. 1 such that the system transfer function $T(j\omega)=FGP/(1+GP)\in\mathfrak{J}$, for all $P\in\mathcal{P}$. One may program the computer to find the (unique) bounds $B(\omega)$ on $G(j\omega)$, so that as P ranges over $\mathcal{P}$,

$$\Delta \ln |T(j\omega)| = \Delta \ln |GP/(1+GP)| \leq [A_2(\omega) - A_1(\omega)]\,\mathrm{dB}$$

Alternatively, this may be done by hand, giving useful insight: the template of $L(j\omega_l)=G(j\omega_l)P(j\omega_l)$ is that of P, shifted (in the Nichols chart) by Arg $G(j\omega_l)$ in the x axis and by 20 $\log_{10}|G(j\omega_l)|$ in the y axis. Suppose $\mathfrak{J}p(j2)$ is given by ABCD in Fig. 7, and one tries positioning it at A′B′C′D′ to give the template of $L(j2)$. From the contours of constant $|L/(1+L)|$ in the Nichols chart, it is seen that the maximum change in $|T(j2)|$ is then closely $(-0.49)-(-5.7)=5.2$ dB, with maximum at C′, minimum at A′. Suppose the $|T(j\omega)|$ tolerance permits a maximum change of 6.5 dB at $\omega=2$, so the above trial is conservative. The template may be shifted lower to A″B″C″D″, at which the $|T(j\omega)|$ tolerances are precisely

Table 1 Loci of six typical lti $P(j\omega)$

	k	ζ	ω_n	V_0	ρ	δ_{max}	α_{max}
a	1	1.0	3.77	180	0.36	06.3	16.4
b	2	1.0	3.77	100	1.05	11.0	23.5
c	4	0.45	5.66	120	1.05	18.6	29.3
d	4	0.45	5.66	117	1.05	22.4	31.3
e	4	1.0	3.77	230	0.36	13.8	27.0
f	3	0.92	7.54	180	0.36	27.4	34.2

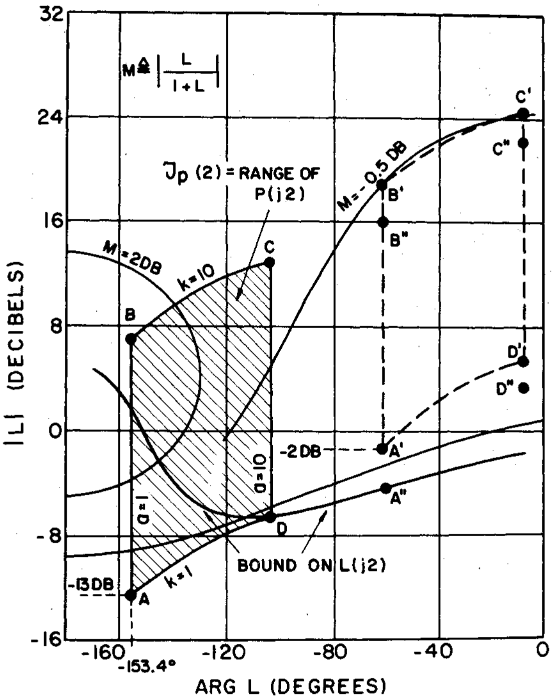


Fig. 7 Hand derivation of bounds on $L(j\omega)$ in Nichols chart.

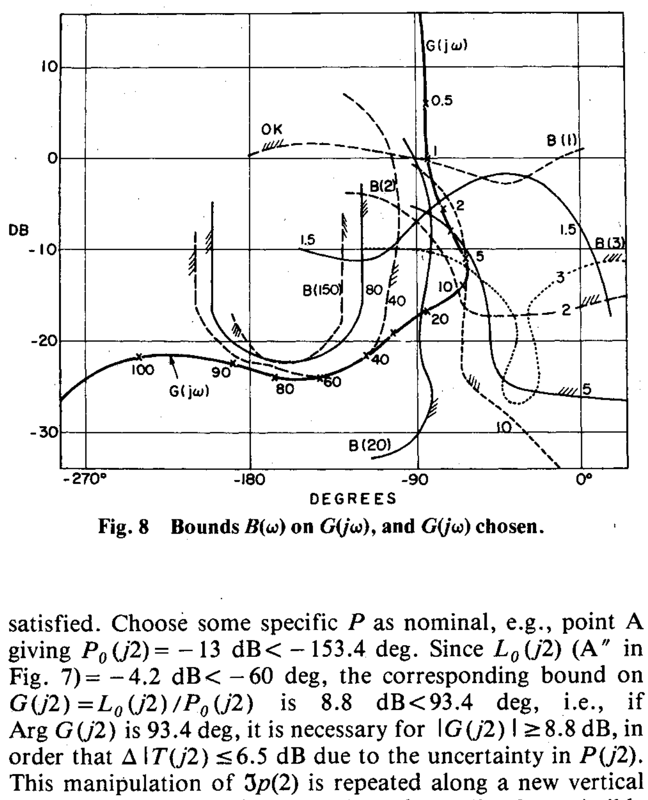


Fig. 8 Bounds $B(\omega)$ on $G(j\omega)$, and $G(j\omega)$ chosen.

satisfied. Choose some specific P as nominal, e.g., point A giving $P_0(j2) = -13$ dB< -153.4 deg. Since $L_0(j2)$ (A″ in Fig. 7)$= -4.2$ dB< -60 deg, the corresponding bound on $G(j2) = L_0(j2)/P_0(j2)$ is 8.8 dB$<$93.4 deg, i.e., if Arg $G(j2)$ is 93.4 deg, it is necessary for $|G(j2)| \geq 8.8$ dB, in order that $\Delta|T(j2)| \leq 6.5$ dB due to the uncertainty in $P(j2)$. This manipulation of $\mathfrak{J}p(2)$ is repeated along a new vertical line, giving another point on the boundary $B(2)$ of permissible $G(j2)$.

Figure 8 shows the bounds so obtained on $G(j\omega)$, and the $G(j\omega)$ chosen to satisfy these bounds. Let e_G be the excess of poles over zeros assigned to $G(s)$, so that as $s \to \infty$, $G \to (k_G/s)^{e_G}$. It is reasonable to define the optimum G as that which satisfies its bounds with minimum k_G. It has been shown[7] that $G_{\rm opt}$ lies on $B(\omega)$ at all ω and that $G_{\rm opt}$ exists and is unique. The design of a practical $G(s)$ to satisfy the bounds is somewhat of an art.[3] For a given skill in the art, the greater the number of poles and zeros of G, the closer one can get to the optimum, so there is tradeoff between complexity and

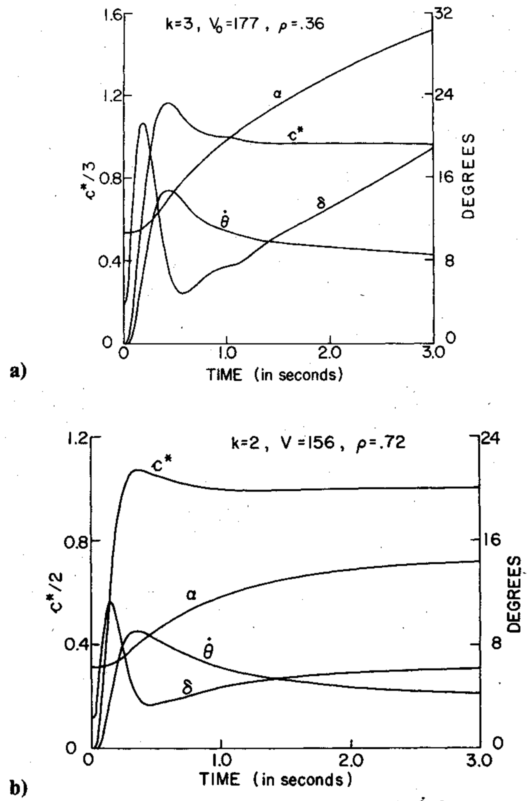


Fig. 9 Representative responses of $\alpha(t)$, $\dot\theta$, δ, and c^*.

bandwidth. Here, we chose simply by cut and try

$$G(s) = (1+0.2s)/s(1+0.033s)^2(1+0.002s+10^{-4}s^2)$$

with very modest bandwidth (Figs. 4 and 8). A much simpler $G(s)$ could have been chosen with larger bandwidth. The designer must make his own tradeoff. Reference 3 offers some advice on the shaping of a function to satisfy a set $\{B(\omega)\}$ in the Nichols chart.

Step 3: Design of $F(s)$

$G(s)$ only guarantees that $\Delta|T(j\omega)| \leq A_2(\omega) - A_1(\omega)$ of Fig. 4, e.g., at $\omega = 10$ the actual change in $|L(j10)/(1+L(j10))|$ is from -7 dB to 4 dB, while from Fig. 4 the permitted change in $|T(j10)| = |FL/(1+L)|$ is from -15 to 2.8 dB. Hence, any value of $|F(j10)| \epsilon [-8, -1.2$ dB$]$ is acceptable. In this way, upper and lower bounds on $|F(j\omega)|$ are obtained and $F(s)$ is chosen to satisfy them, which is also somewhat of an art. In this example (Fig. 4), a satisfactory $F(s)$ is

$$F(s) = (1+0.33s)(1+0.05s)/(1+0.25s)(1+0.2s)(1+0.0125s)^2$$

Design Results

The nonlinear system was simulated and its response found for several hundred command inputs and gust disturbances, some of which are shown in Figs. 3 and 9-12. Some typical responses to c^* step commands are shown in Fig. 3 for various V_0, ρ, and step (k) values. The assigned bounds on c^* are also shown. The transient responses of $\alpha(t)$, $\dot\theta(t)$ etc. depend, of course, on the values of k, ρ, and V_0. Two sets of these are shown in Figs. 9a and 9b, with Fig. 9a depicting very large $\alpha(t)$ excursion, for which the $C_{ij}(\alpha)$ in Fig. 2 are in strongly nonlinear ranges. These are the inputs for which the system was designed and for which guarantees can be made. In a very few cases there was very slight excursion out of the bounds. It is possible to include in the design other inputs and gust disturbances, with specified response tolerances, and

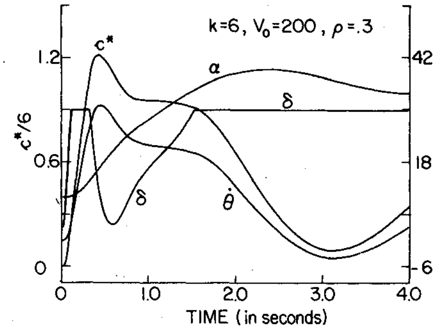
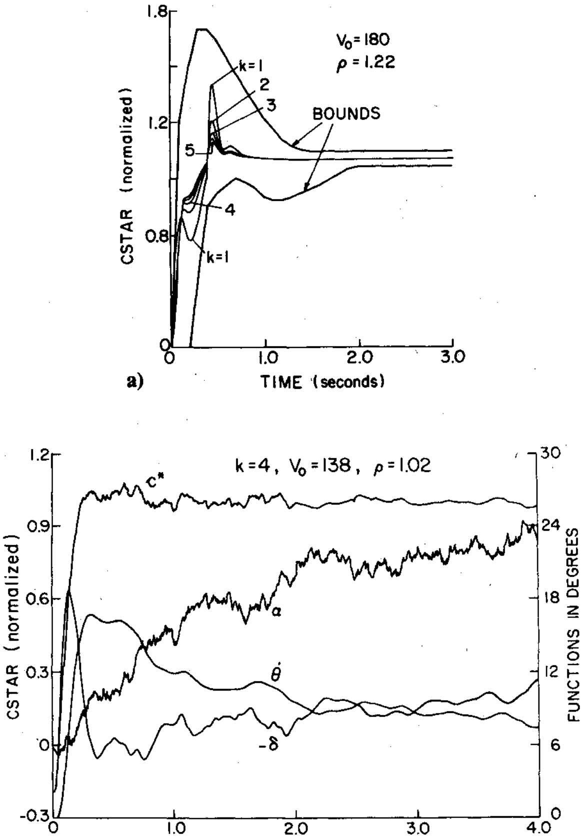


Fig. 10 Responses for input causing hard δ saturation.



a)

b)

Fig. 11 Responses to gusts and simultaneous c* step command: a) half-sine gust, b) random Gaussian.

then guarantees can be made for these as well. The response to other inputs is nevertheless also found to be quite satisfactory here. This is typical of the design, i.e., the system is not very sharply tuned to the class of inputs used in the design execution. There is reasonable response continuity to other inputs.

Some responses to very large c^* step commands causing hard δ saturation are shown in Fig. 10. For the response to gust disturbances, the gust input was modeled by replacing α in Eq. (3) by $\alpha=\tan^{-1}(u/V_0)+\alpha_{gust}$. Two kinds of α_{gust} were used. In one α_{gust} is a half-sine wave of amplitude $20/V_0$ rad and half-period THALF $\epsilon[0.2,2]$s. Some results are shown in Figs. 11a, b. In both, the gust begins precisely at the instant of application of simultaneous c^* step commands. The second kind, as in Fig. 11b, is stochastic guassian with power spectrum $k/(1+\omega^2)V_0^2$ and $(\alpha_{gust})_{rms}=6/V_0$ rad. Examples of responses to a single square wave c^* command with equal positive and negative values k and total duration 2 THALF are shown in Fig. 12.

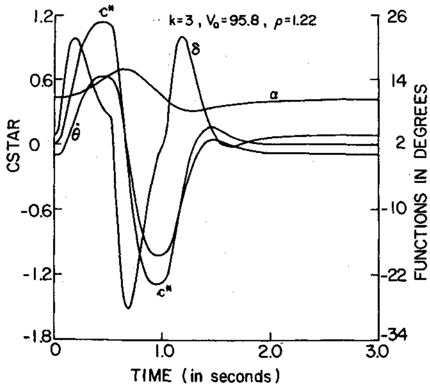


Fig. 12 Responses to square wave c* command.

Discussion and Conclusions

A second-order model was used for $T(s)$ with excess of poles over zeros $e_T=1$, in step. 1. This appears to be incompatible with G, F fifth order and excesses $e_G=4$, $e_F=2$, and P with $e_p=0$. [No analytical expression but linearized Eqs. (1-4) give second-order P.] Strict design execution appears to require a $T(s)$ of complexity compatible with $T=FGP/(1+GP)$. Note, however, that in step 1, the model of $T(s)$ is used only to generate a set which covers the range of a priori specified acceptable outputs $\mathcal{Y}$. Any $T(s)$ model which achieves this is clearly satisfactory, and the simpler the better. The designer can later choose the complexity of $G(s)$, $F(s)$, with no regard for that of the $T(s)$ model used in step 1.

The class of applicable nonlinearities has been defined implicitly in Ref. 2, but one very large class can be defined explicitly. Let $D_1y(t)=D_2x(t)$ with D_1, D_2 operators which may be nonlinear, uncertain, and time-varying, e.g.,

$$D_1y=M(\ddot{y})(\dot{y})^{0.5}\mathrm{sgn}\dot{y}+\left(\frac{A+Bte^{-\alpha t}}{E+Ft}\right)\dot{y}^2|y|^n+Hy^2$$

$$E\epsilon[1,5]\quad F\epsilon[0.5,4]\quad A\epsilon[-3,6]\quad \alpha\epsilon[0.5,1.5]$$

$$B\epsilon[-3,2]\quad H\epsilon[-4,1]\quad n\epsilon[0.5,2]\quad M\epsilon[1,5]$$

The range of M must be of the same sign. All $y\epsilon\mathcal{Y}$ and D_1y must be bounded for all t in $[0,\infty]$ and D_1y must exist. Hence y must be twice differentiable except, at most, at a countable number of points. Then $D_1y\triangleq\psi(t)$ is known and there must exist a unique solution for x in $D_2x=\psi(t)$. The solution must be bounded for all $t\epsilon[0,\infty]$. Thus, $D_2x=\psi(t)$ must be "bounded-input, bounded-output" stable. However, $D_1z(t)=\nu(t)$ may be "unstable" in that a bounded $\nu(t)$ is allowed to result in unbounded $z(t)$. It is only necessary that bounded $z(t)$ gives bounded $\nu(t)$.

It is possible that a simple linearization might do just as well in practice, but this is a matter of chance, whereas this design technique is guaranteed to work if the constraints are satisfied. It can be argued that the constraint of uniqueness may be waived in the design technique if the set of inputs $\{x(t)\}$ which can give the single output $y(t)$, is "compact," i.e. if that set can be covered as accurately as desired by a finite number of elements. A single w, y pair then generates a set of lti P instead of only one. However, this has not been rigorously proven as yet.

The constraint on $\mathcal{W}$ that $\mathcal{P}$ is mp, is required because only then can one guarantee that *any* specifications no matter how narrow (A_2-A_1 arbitrarily small but nonzero in Fig. 4), may be satisfied for arbitrarily large but bounded parameter uncertainty (but some parameters must not change sign[7]). No such guarantee can be made for nmp $\mathcal{P}$, but the problem is still solvable if the specifications are not too narrow and $\mathcal{P}$ is not too large[8,9] a set.

Finally, a recent extension[7] is to linear and nonlinear multiple input-output systems, where the problem is transformed into the design of n^2 single-loop lti systems like Fig. 1.

Acknowedgments

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References

[1] Garrard, W.L. and Jordan, J.M., "Design of Nonlinear Automatic Flight Control Systems," *Automatica,* Vol. 13, Sept. 1977, pp. 497-505.

[2] Horowitz, I., "Synthesis of Feedback Systems with Nonlinear Time-Varying Plants to Satisfy Quantitative Performance Specifications," *Proceedings of the IEEE,* Vol. 64, Jan. 1976, pp. 123-130.

[3] Horowitz, I. and Sidi, M., "Synthesis of Feedback Systems with Large Plant Ignorance for Prescribed Time-Domain Tolerances," *International Journal of Control,* Vol. 16, Aug. 1972, pp. 287-309.

[4] Toble, H., Elliott, E., and Malcolm, L., "A New Longitudinal Handling Qualities Criterion," The Boeing Co., Commercial Airplane Division, Report TL 693.N3, May 1966.

[5] Kisslinger, L., et al., "Survivable Flight Control System," Interim Report 1, McDonnell Douglas, May 1971, p. 12, Table 1.

[6] Laacke, F. and Miller, C., "Comparison of the Basic F4E and the Slotted F4E Stall/Near Stall/Spin Characteristics," McDonnell Douglas Rept. MDC A 2181, Dec. 1969.

[7] Horowitz, I., "Quantitative Synthesis of Uncertain Multiple Input-Output Feedback Systems," *International Journal of Control,* Vol. 30, No. 1, 1979, pp. 81-106.

[8] Horowitz, I. and Sidi, M., "Optimum Synthesis of Nonminimum-Phase Feedback Systems with Plant Uncertainty," *International Journal of Control,* Vol. 27, March 1978, pp. 361-386.

[9] Horowitz, I., *Synthesis of Feedback Systems,* Academic Press, New York, 1963, Chap. 7.